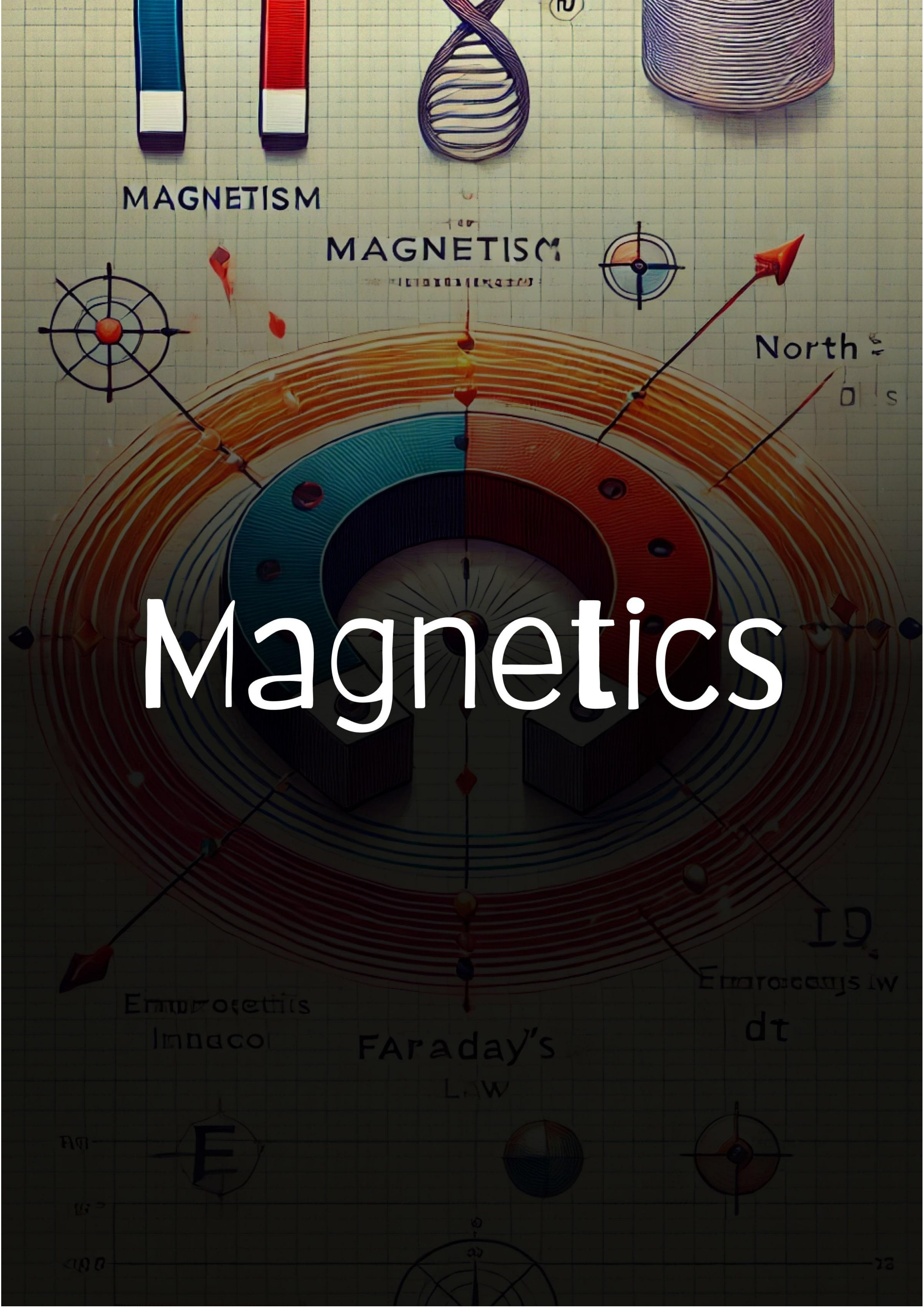
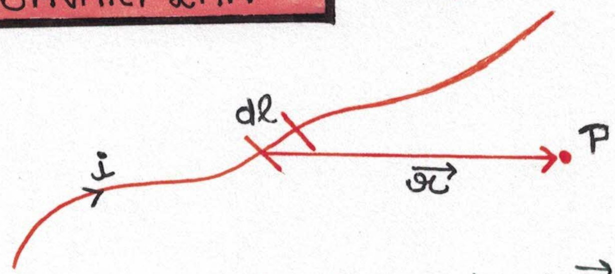


Magnetics

Magnetics

BIOT & SAVART LAW



Consider a current element $i\vec{dl}$ in a current carrying wire for which we want to find the magnetic field at a point P whose position motor with respect to the element is $\vec{r}$, then the small amount of magnetic field produced by the current element at point P is given by

$$\left(d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i\vec{dl} \times \vec{r}}{r^3}\right)$$

BIOT & SAVART LAW IN TERMS OF CHARGE & DRIFT VELOCITY

Let dq amount of charge flow through in time interval dt.

Then $\quad i = \dfrac{dq}{dt} \quad , \quad \vec{V_d} = \dfrac{d\vec{l}}{dt}$

$$\therefore d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{dq}{dt} \cdot \vec{V_A}\, dt \cdot \frac{\vec{r}}{r^3}$$

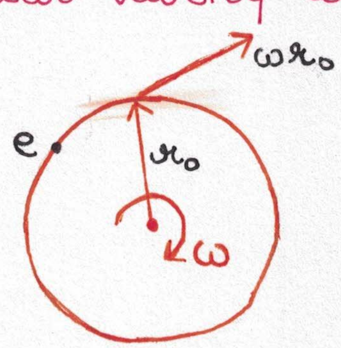
$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot dq \cdot \frac{\vec{V_d} \times \vec{r}}{r^3}$$

FIELD DUE TO A POINT CHARGE

$$\left(\vec{B} = \frac{\mu_0}{4\pi} \cdot q\, \frac{\vec{v} \times \vec{r}}{r^3}\right)$$

Que.) Find magnetic field at the center of a hydrogen atom assuming electron revolves with angular velocity ω & radius is r_0.

$\vec{V} \times \vec{r}$ into the plane



$$B = \frac{\mu_0}{4\pi}\, e\, \frac{\omega r_0\, r_0 \sin 90^\circ}{r_0^3} \odot$$

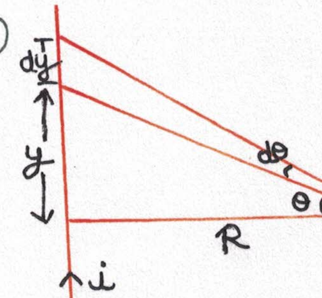
out of the board

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{e\omega}{r_0} \odot$$

Que.) Derive an expression for magnetic field due to a finite & an infinite straight current carrying thin wire.

$$\vec{dB} = \frac{\mu_0}{4\pi}\, i\,dy \cdot \frac{R\sec\theta \cdot \sin(90+\theta)}{(R\sec\theta)^3} \otimes$$

(into plane)



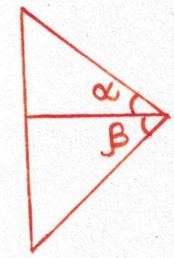
$y = R\tan\theta$

$dy = R\sec^2\theta . d\theta$

$$dB = \frac{\mu_0}{4\pi}\, \frac{i R^2 \cancel{\sec^3}\theta . d\theta . \cos\theta}{R^3 \cancel{\sec^3\theta}}$$

$$dB = \frac{\mu_0}{4\pi}\, i \frac{\cos\theta\, d\theta}{R}$$

($\alpha \rightarrow$ upper extremity (in direction of current)
$\beta \rightarrow$ against the current)

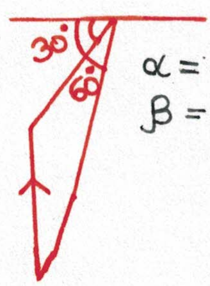


For finite wire $\quad B = \dfrac{\mu_0 i}{4\pi R}\,[\sin\alpha + \sin\beta]$

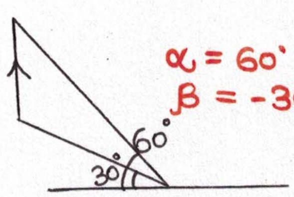
$$\alpha = \beta = \frac{\pi}{2}$$

For infinite wire $\quad B = \dfrac{\mu_0 i}{2\pi R}$

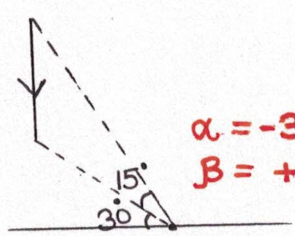
Eg:-



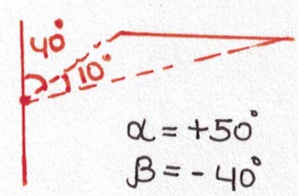
$\alpha = -30°$
$\beta = +60°$



$\alpha = 60°$
$\beta = -30°$



$\alpha = -30°$
$\beta = +45°$



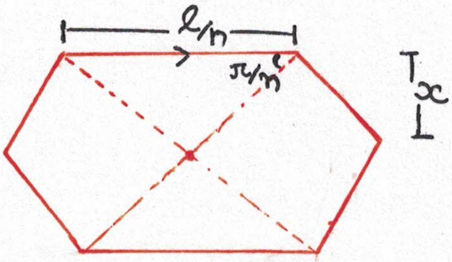
$\alpha = +50°$
$\beta = -40°$

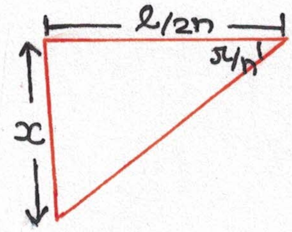
Que.) You have a wire of length 'l' which you convert to a regular polygon loop of 'n' sides.

(a) What is the field at the center of polygon.

(b) Can you use this formula to find field at center of circle. If yes, find the field due to circle. Express answer in terms of radius. Take current in polygon to be 'i'.

NOTE : For a straight conductor, point the thumb of your right hand in the direction of current. Now the curl of the fingers gives the sense of concentric field lines.





$$\frac{\frac{l}{2n}}{x} = \tan\frac{\pi}{n}$$

$$x = \frac{l}{2n\tan(\pi/n)}$$

$$B_1 = \frac{\mu_0 i}{4\pi x}\left[\sin(\pi/n) + \sin(\pi/n)\right]$$

$$= \frac{2\mu_0 i n}{4\pi l}\tan\pi/n \cdot 2\sin\pi/n$$

$$\left(B = nB_1 = \frac{\mu_0 i}{\pi l} n^2 \tan\frac{\pi}{n}\cdot\sin\frac{\pi}{n}\right)$$

For circle, $n \to \infty$

as $n \to \infty$, $\frac{\theta}{n} \to 0$ $\quad\therefore \tan\frac{\pi}{n} \approx \sin\frac{\pi}{n} \approx \frac{\pi}{n}$

$$B = \frac{\mu_0 i}{\pi l}\cdot \cancel{n^2}\cdot\frac{\pi}{\cancel{n}}\cdot\frac{\pi}{\cancel{n}}$$

$$B = \frac{\mu_0 i \pi}{l}$$

$$B = \frac{\mu_0 \pi i}{2\pi R}$$

$$\left(B = \frac{\mu_0 i}{2R}\right) \otimes$$

FIELD DUE TO ARC OF A CIRCLE AT THE CENTRE OF CURVATURE

Since we know the field due to a circle, we can find the field due to arc by taking its proportionate contribution.
i.e. Mathematically,

$$B_{arc} = B_{circle} \times \frac{\theta}{2\pi}$$

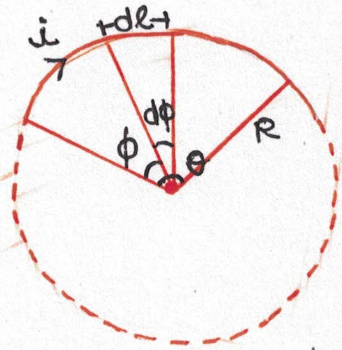
$$\left(B_{arc} = \frac{\mu_0 i}{4\pi R} \cdot \theta \right)$$

FIELD OF AN ARC BY INTEGRATION

$$dB = \frac{\mu_0}{4\pi} \frac{i R d\phi \sin 90^\circ \times R}{R^3}$$

$$\int_0^B dB = \frac{\mu_0 i}{4\pi R} \int_0^\theta d\phi$$

$$B = \frac{\mu_0 i}{4\pi R} \cdot \theta$$



$(dl = R d\phi)$

For circle, $\theta = 2\pi$

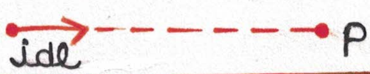
$$\therefore B_{circle} = \frac{\mu_0 i}{2R}$$

NOTE: 1.) For finding the direction of field at the center of curvature of circular arcs, curl the fingers of your right hand in the sense of current. Now the thumb gives the direction of magnetic field at the center of curvature.
(The same technique can also be used for finding magnetic field direction in the region enclosed in any convex loop.)

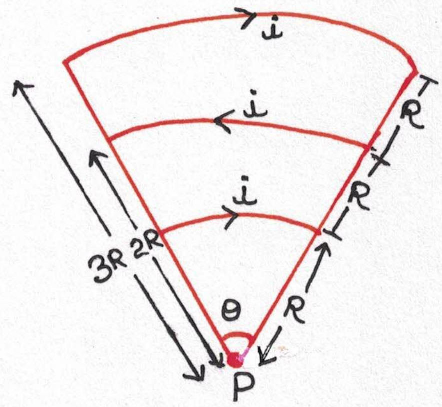
Eg : In a square.

2.) If a point lies on the extension of a straight segment of a wire then the contribution to the magnetic field by that segment is zero.

Eg: Contri. of idl towards magnetic field at P=0



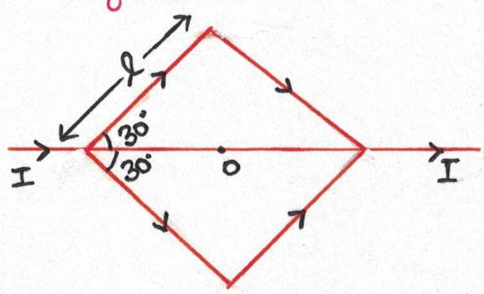
Que.) Find net magnetic field at point P. Take inside the plane to be the positive.



$$B_P = \frac{\mu_0 i \theta}{4\pi R}\left[1 - \frac{1}{2} + \frac{1}{3}\right]$$

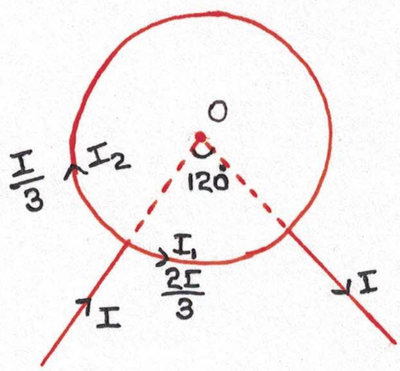
$$B = \frac{5\mu_0 i \theta}{24\pi R} \otimes$$

Que.) Find field at O



$$B_o = \text{zero}$$

Que.) Find field at zero.



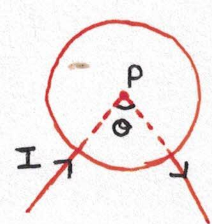
$$B_1 = \frac{\mu_0}{4\pi}\,\frac{2IR}{3}\,\frac{(120^\circ).\,R\sin 90^\circ}{R^3} = \frac{\mu_0}{\pi} \times \frac{120}{R}$$

$$B_2 = -\frac{\mu_0}{4\pi}\,\frac{I}{3}\,\frac{R(240^\circ)R\sin 90^\circ}{R^3} = \frac{-\mu_0}{120\pi}\cdot\frac{240}{R}$$

$$B_1 - B_2 = \frac{\mu_0}{\pi R}\cdot\left(\frac{120}{6} - \frac{240}{12}\right)$$

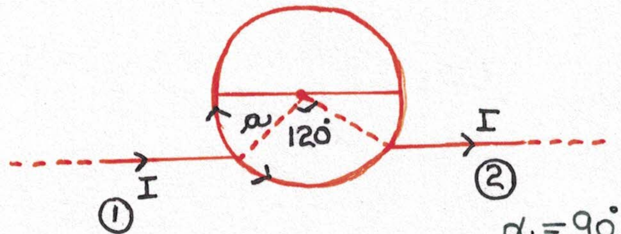
$$B_1 - B_2 = 0$$

$$\therefore\ B_o = 0 = \text{zero}$$



In general,
(Magnetic field at P = 0)

Que.) Find field at center.



① $\alpha = -60°$, $\beta = 90°$

② $\alpha = 90°$, $\beta = -60°$

$$B_1 = \frac{\mu_0 I}{4\pi R}\left(1-\frac{\sqrt{3}}{2}\right)$$

$$B_2 = \frac{\mu_0 I}{4\pi R}\left(1-\frac{\sqrt{3}}{2}\right)$$

$$\therefore B = B_1 + B_2 = \frac{\mu_0 I}{2\pi R}\left(1-\frac{\sqrt{3}}{2}\right)$$

FIELD DUE TO A THIN COIL OF 'N' CURLS OR TURNS

$$\left(B = \frac{\mu_0 N i}{2R}\right)$$

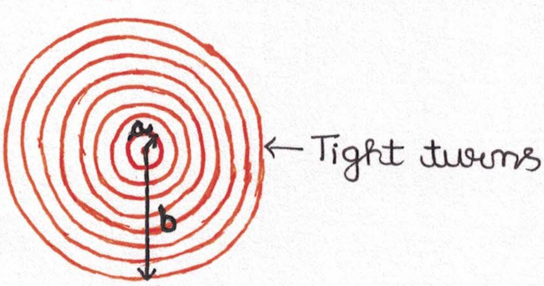
Que.) The field at the center of a coil of single turn is B. If now the same wire is converted to a coil of 2 turns, what will be the new field, keeping the current same?

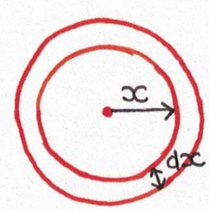
$$B_{New} = 4B$$

$\because N_0 = 1$, $R_0 = R$

$N_n = 2$, $R_n = R/2$

Que.) Find field at center of coil given that current is I.





$$dB = \frac{\mu_0 i}{2x} \cdot \frac{dx}{D}$$ ← no. of turns in thickness dx

D = diameter of wire

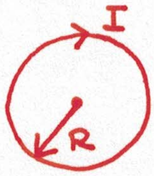
$$\int_0^B dB = \frac{\mu_0 i}{2D}\left[\ln x\right]_a^b$$

$$B = \frac{\mu_0 i}{2D} \ln\left(b/a\right)$$

MAGNETIC MOMENT OF A CURRENT LOOP

The magnetic moment of a current loop is a vector quantity whose magnitude is equal to current times the area enclosed by the loop & the direction is given by 'Right Hand Thumb Rule'. Curl the fingers of your right hand in the sense of the current. Now the thumb gives the direction of magnetic moment.

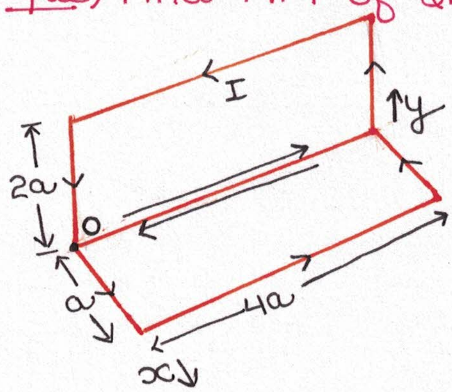
For eg:



$$\left(\vec{M} = I\pi R^2 \otimes\right)$$

(For 3-D loops, we find M.M. by breaking it into a combo of 2-D loops.)

Eg: Que.) Find MM of shown figure.

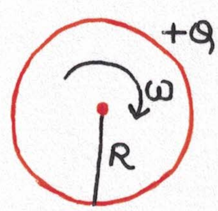


$$\vec{M} = I \times 8a^2\,\hat{i} + I \times 4a^2\,\hat{j}$$

$$\vec{M} = 4Ia^2\,[2\hat{i} + \hat{j}]$$

$$|\vec{M}| = 4\sqrt{5}\,Ia^2$$

Que.) Find MM of the ring carrying a uniformly distributed charge $(+Q)$.



$$I = \frac{Q}{2\pi/\omega} = \frac{Q\omega}{2\pi}$$

$$M = IA$$

$$= \frac{Q\omega}{2\pi}\cdot \pi R^2$$

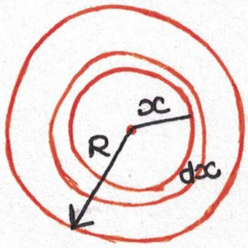
$$\vec{M} = \frac{Q\omega R^2}{2} \otimes$$

Que.) For previous problem find (angular momentum : MM) Take mass $= m$.

$$I = mR^2 \Rightarrow I\omega = m\omega R^2$$

$$AM : MM = \frac{m\omega R^2}{\frac{Q\omega R^2}{2}} = \frac{2m}{Q}$$

Que.) Find angular Momentum : Magnetic Momentum of a uniformly charged homogenous disc having mass m & charge Q.



$$dM = dq \cdot \frac{\omega x^2}{2} = ?$$

$$\int_0^M dm = \frac{Q\omega}{R^2}\int_0^R x^3 \cdot dx$$

$$dq = \frac{Q}{\pi R^2} \cdot 2\pi x\,dx$$

$$M = \frac{Q\omega}{4R^2} \cdot R^4$$

$$dq = \frac{2Q}{R^2} \cdot x\,dx$$

$$M = \frac{Q\omega R^2}{4}$$

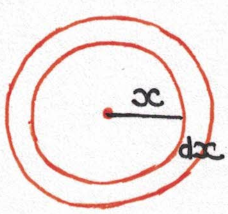
$$AM : MM = \frac{\frac{1}{2}M\omega R^2}{\frac{Q\omega R^2}{4}} = \frac{2M}{Q}$$

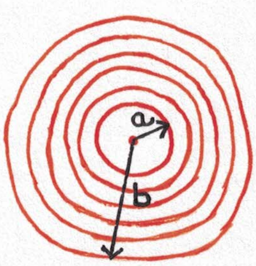
NOTE: For any shape body,

$$\frac{\text{angular Momentum}}{\text{magnetic Momentum}} = \frac{2M}{Q}$$

Que.) Find M.M. of a tightly bound spiral in a previous problem.

D = diameter of wire

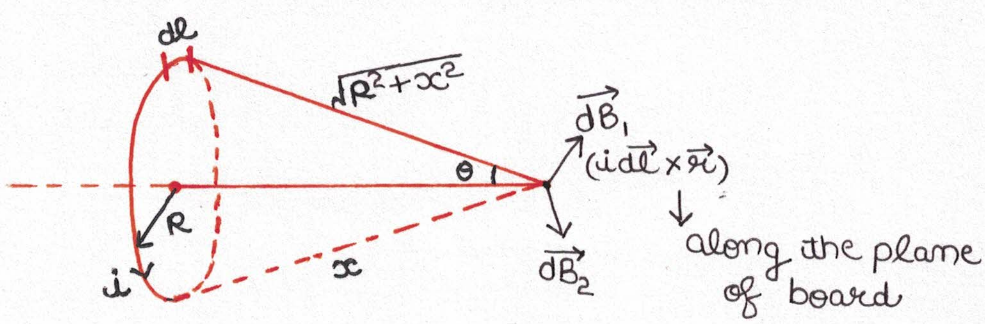




$$\int dM = \int_a^b I\pi x^2 \frac{dx}{D}$$

$$M = \frac{I\pi}{3D}\left[b^3 - a^3\right]$$

FIELD ON THE AXIS OF A CURRENT CARRYING LOOP



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$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,d\vec{l} \times \vec{r}}{r^3}$$

(Radial components cancel out & axial components add up)

$$dB = \frac{\mu_0}{4\pi} \frac{i\,dl\sqrt{R^2+x^2}\cdot \sin 90^\circ}{(\sqrt{R^2+x^2})^3}$$

$$dB_{axial} = dB\sin\theta = \frac{\mu_0}{4\pi} \frac{i\,dl\sqrt{R^2+x^2}}{(\sqrt{R^2+x^2})^3} \cdot \frac{R}{\sqrt{R^2+x^2}}$$

$$dB_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{i\,dl.R}{(R^2+x^2)^{3/2}}$$

$$B_{axial} = \frac{\mu_0}{4\pi} \frac{i\,dl.R}{(R^2+x^2)^{3/2}}$$

$$B_{axial} = \frac{\mu_0}{4\pi} \frac{iR}{(R^2+x^2)^{3/2}} \int dl$$

$$B_{axial} = \frac{\mu_0}{4\cancel{\pi}} \cdot \frac{iR\,2\cancel{\pi}R}{(R^2+x^2)^{3/2}} \qquad (\because x, R \rightarrow \text{Constant})$$

$$B_{axial} = \frac{i\mu_0 R^2}{2(R^2+x^2)^{3/2}} \qquad \text{Now, } i\pi R^2 = M$$

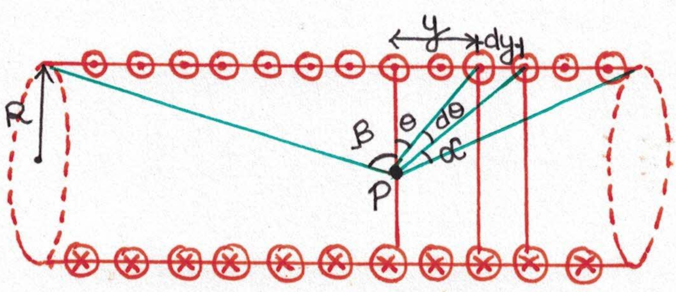
$$\therefore \left(\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{M}}{(R^2+x^2)^{3/2}}\right)$$

If $x >> R$, $\vec{B} = \frac{\mu_0}{4\pi} \frac{2\vec{M}}{x^3}$

which is similar to, $\left(\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2\vec{p}}{x^3}\right)$

$\downarrow$ dipole axis

FIELD DUE TO A SOLENOID



$n \rightarrow$ no. of turns per unit length

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{2\pi R^2 i}{(R^2+y^2)^{3/2}} \cdot n\,dy$$

Now we know, $y = R\tan\theta$

$dy = R\sec^2\theta . d\theta$

Now, $(R^2+y^2)^{3/2} = R^3\sec^3\theta$

$$\therefore \int dB = \frac{\mu_0}{4\pi} \cdot \frac{2\pi R^2 i n}{R^3} \int \frac{d\theta}{\sec^3\theta} \cdot R\sec^2\theta$$

$$\int dB = \frac{\mu_0}{4\pi} \cdot 2\pi i n \int_{-\beta}^{\alpha} \cos\theta . d\theta$$

$$\int dB = \frac{\mu_0 i n}{2} \left[\sin\theta\right]_{-\beta}^{\alpha}$$

$$\left(B = \frac{\mu_0 n i}{2} \left[\sin\alpha + \sin\beta\right] \right)$$

Special Case:

1.) P at interior of infinite Solenoid OR For infinite Solenoid, $(\alpha = \beta = \pi/2)$

$$\therefore B = \mu_0 n i$$

2.) P near end point of infinite solenoid OR at end point of infinite solenoid, $(\alpha = 0,\ \beta = \pi/2)$

$$\therefore B = \frac{1}{2}\mu_0 n i$$

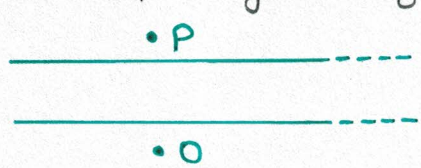
NOTE: 1.) Field at end points $= \frac{1}{2}$ (Field at interior point)

2.) For an infinite solenoid, field at any point within its volume is $\mu_0 n i$. and direction is along the axis.

(Both will be proved later by Ampere's Law).

3.) For an outside point OR a point outside the infinite solenoid, Magnetic field is zero.

Eg:



$B_P = B_O = 0$

AMPERE'S LAW

The line integral of magnetic field for any closed amperial loop around a wire carrying a steady current is 'μ_0' times the current enclosed by the loop.

Mathematically,

$$\left(\oint \vec{B}.\vec{dl} = \mu_0 i_{enclosed}\right)$$

Choose any sense of amperian loop for the direction of 'dl'. Now curl the fingers of your right hand in the sense of the amperian loop.

→ If a current crosses the loop in this direction, it will be taken as +ve, whereas

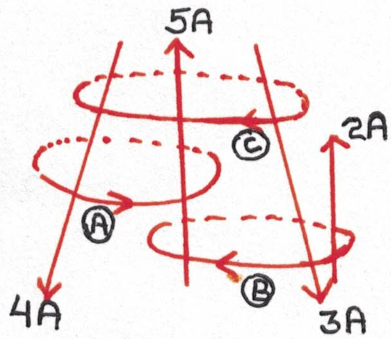
→ if it crosses in the opposite direction, it will be taken as -ve.

Que.) Find the current enclosed for various Amperian loops shown in the fig.

Ⓐ → +1A

Ⓑ → -4A

Ⓒ → +2A



Que.) Verify the Ampere's law for a circular loop with an infinite current carrying wire at its axis.

We need to prove

$$\int \vec{B}.\vec{dl} = \mu_0 i$$

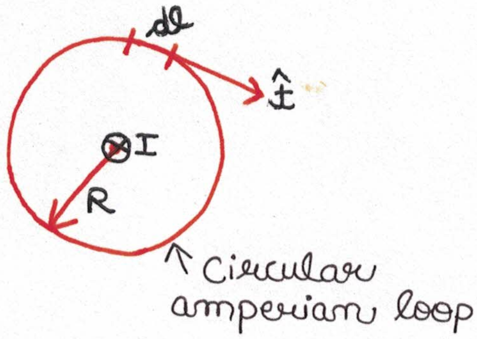
$$\vec{B} = \frac{\mu_0 i}{2\pi R}\,\hat{i}$$

$$\vec{dl} = dl\,\hat{i}$$

$$\vec{B}.\vec{dl} = \frac{\mu_0 i}{2\pi R}\,dl$$

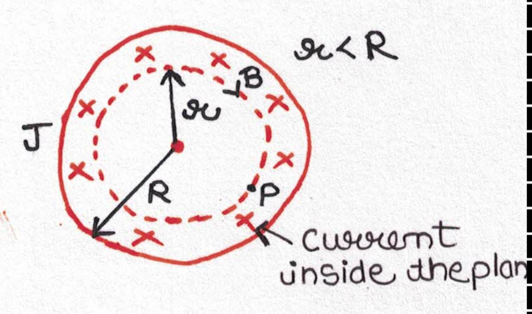
$$\int \vec{B}.\vec{dl} = \frac{\mu_0 i}{2\pi R}\oint dl = \mu_0 i$$

$$\left(\because \int dl = 2\pi R\right)$$



Que:) Use Ampere's law to derive magnetic field due to an infinite cylinder having a uniform current density along its axis.

$r < R$

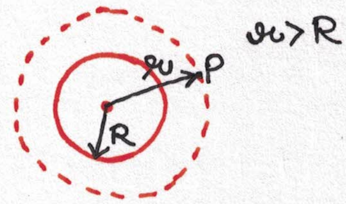


$$\int \vec{B} \cdot d\vec{l} = \mu_0 i_{enclosed}$$

$$B \times 2\cancel{\pi}\cancel{r} = \mu_0 J \cancel{\pi} r^{\cancel{2}}$$

$$B = \frac{\mu_0 J r}{2}$$

$$\vec{B} = \mu_0 \frac{\vec{J} \times \vec{r}}{2}$$

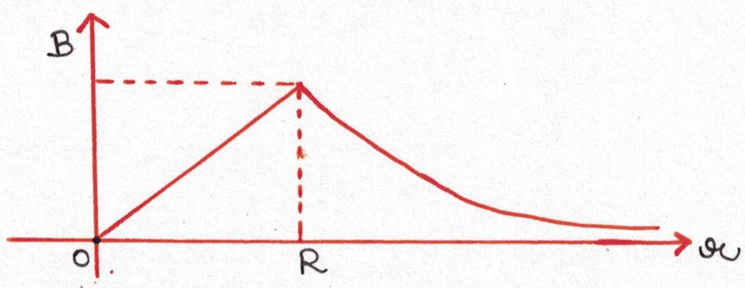


$r > R$

$$B \times 2\pi r = \mu_0 J \pi R^2$$

$$B = \frac{\mu_0 J R^2}{2r} ,$$

$$\vec{B} = \frac{\mu_0 J R^2}{2r} (\widehat{\vec{J} \times \vec{r}})$$

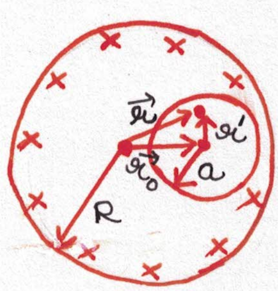


Que:) An infinite cylinder having a uniform current density 'J' along its axis has a cavity of radius 'a' parallel to its axis. Show that magnetic field in the cavity is uniform. Also find its magnitude & direction.

$$\vec{B}_P = \frac{\mu_0 \vec{J} \times \vec{r}}{2} - \frac{\mu_0 \vec{J} \times \vec{r'}}{2}$$

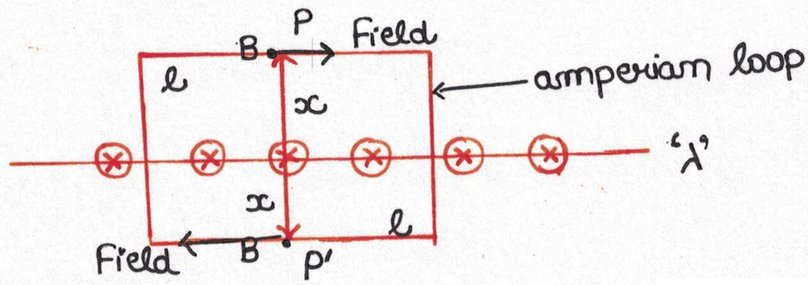
$$= \frac{\mu_0 \vec{J}}{2} \times (\vec{r} - \vec{r'})$$

$$B = \frac{\mu_0 \vec{J} \times \vec{r_0}}{2}$$



uniform field downwards

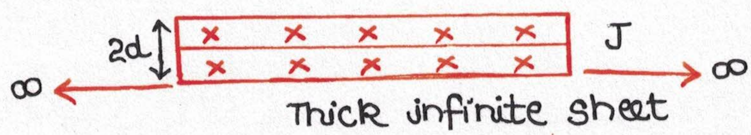
Que.) Find the magnetic field due to an infinite sheet of current carrying linear current density 'λ' along its length.



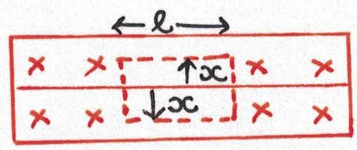
$$2Bl = \mu_0 \lambda l$$

$$\left(B = \frac{\mu_0 \lambda}{2}\right)$$

Que.) Find magnetic field as a function of distance 'x' from the bisecting plane of the sheet.



Case I, $x < d$



$$2Bl = \mu_0 J \times 2xl$$

$$B = \mu_0 J x$$

Case II, $x > d$

$$2Bl = \mu_0 \cdot 2dlJ$$

$$B = \mu_0 J d$$

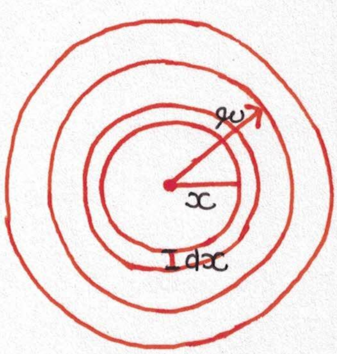
Que.) Magnetic field in a cylindrical region varies as $(B = h \cdot r^{\alpha})$. Find J as a function of r.

Let $J = K \cdot (r)^n$

$$b \cdot r^{\alpha} l = \mu_0 K r^n 2\pi r l$$

$$b \cdot r^{\alpha} = \mu_0 \cdot 2\pi K r^{n+1}$$

$$\int di_{enc} = \int_0^r kx^n \cdot 2\pi x \, dx$$



$$B \times 2\pi r = 2\pi K \cdot \frac{r^{n+2}}{n+2}$$

$$B = b r^{\alpha+1} = \mu_0 K \frac{r^{n+2}}{n+2}$$

$$K = \frac{b(n+2)}{\mu_0} \quad , \quad \alpha + 1 = n+2$$

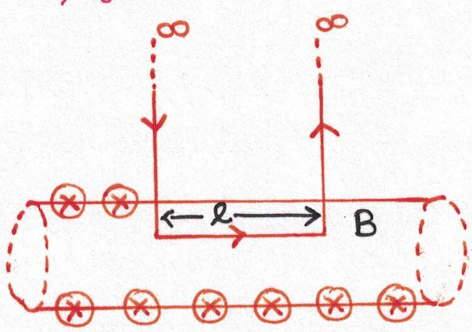
$$\alpha = n+1$$

$$n = \alpha - 1$$

$$J = \frac{(n+2)b}{\mu_0} \cdot r^{\alpha-1}$$

$$J = \frac{(\alpha+1)b}{\mu_0} r^{\alpha-1}$$

Que.) Using Ampere's law show that field inside an infinite Solenoid is '$\mu_0 n i$'.



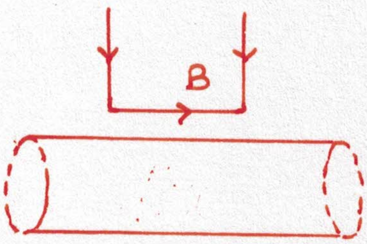
$$Bl = \mu_0 n i l$$

$$B_{in} = \mu_0 n i$$

For outside point,

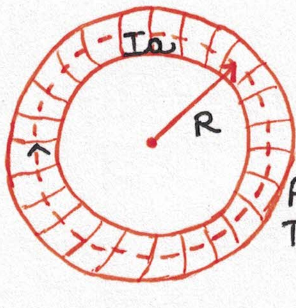
$$Bl = \mu_0 \times \underset{\downarrow}{0}$$

$i_{enclosed}$



$$\therefore B_{out} = 0$$

FIELD INSIDE TOROID



$a << R$

Field Tangential

$$B_{outside} = 0$$

$$B_{inside} = \mu_0 n i$$

$$\underbrace{2\pi R n i}_{\text{Current Crossing loop}} = B \times 2\pi R$$

MAGNETIC FORCE ON A CHARGED PARTICLE

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

where, q = charge on particle
v = velocity of particle
B = local magnetic field

NOTE: Some conclusions that can be drawn from this formulae:

1.) A particle at rest experiences no magnetic force.
2.) A particle moving parallel or anti-parallel to $\vec{B}$ experiences no magnetic field.
3.) Since $\vec{v} \perp \vec{F}$, $\therefore P = 0 \rightarrow \frac{dw}{dx} = 0$

ie. the magnetic field does no work on a moving charged particle ($\vec{v} \perp \vec{F}$) & therefore the kinetic energy of the particle remains conserved. Speed will be constant.

LORENTZ FORCE

The total electro-magnetic force experienced by a particle in an electro-magnetic field is called Lorentz Force.
(Sum of electric and magnetic field is same in all interval frames.)

Mathematically, $\left[\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})\right]$

Since velocity is frame dependent, therefore magnetic force is also frame dependent.

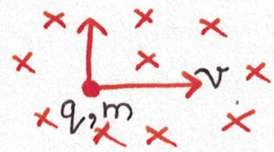
In one inertial frame, we see acceleration of particle & in second inertial frame, we observe same acceleration. So force is same but mag. force has changed as velocity has changed. Therefore

Force (net) = Electric + Magnetic
(and is constant)

When Magnetic force ↑, electric force ↓ &
When Magnetic force ↓, electric force ↑.

Case I

$E = 0$ & $v \perp B$



(Let mag. field be inside the board)

$$F_{centripetal} = qvB = \frac{mv^2}{R}$$ (R = instantaneous radius of curvature

$$\left(R = \frac{mv}{qB}\right)$$

R is constant ∵ v is constant, since mag. field does no work.

∴ Path is circle

$$\omega = \frac{v}{R}$$

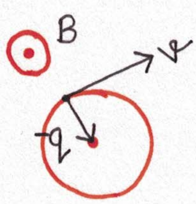
$$\left(\omega = \frac{qB}{m}\right)$$

$$T = \frac{2\pi}{\omega}$$

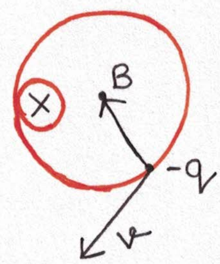
$$\left(T = \frac{2\pi m}{qB}\right)$$

Que.) Sketch/Draw the path taken by particle in following cases :

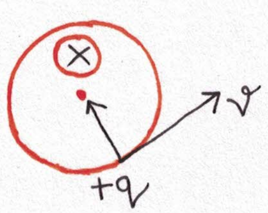
(a)



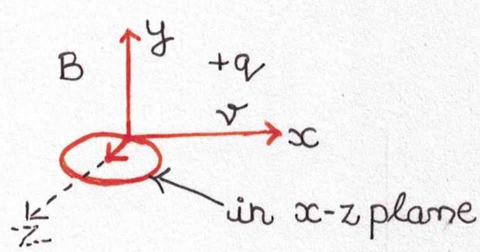
(b)



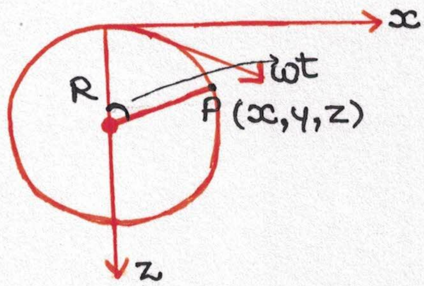
(c)



(d)



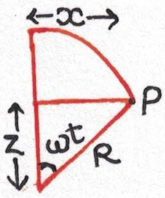
Que.) A particle of mass 'm' & Charge 'q' is launched with a speed 'v' $\hat{i}$ in a region of constant magnetic field 'B' $\hat{j}$ from the origin. Find the coordinates of the particle as a function of time.



Distance in time 't'

$= \omega t$

$= \dfrac{qBt}{m}$

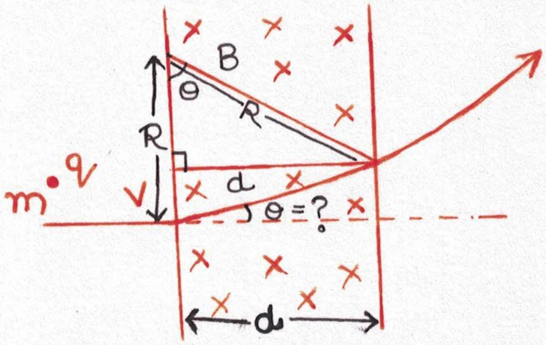


$x = R \sin \omega t$

$z = R[1 - \cos \omega t]$

$y = 0$

Que.) A particle is launched in a strip having uniform magnetic field as shown in the fig. Find the angle of deviation of the particle in terms of 'B', 'q', 'm', 'v' and 'd'.



$F = qvB$

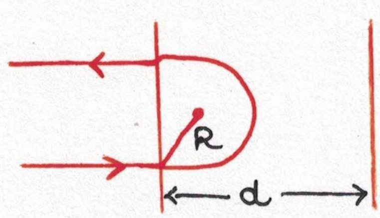
$\sin\theta = \dfrac{d}{R}$

$\theta = \sin^{-1}\left(\dfrac{d}{R}\right)$

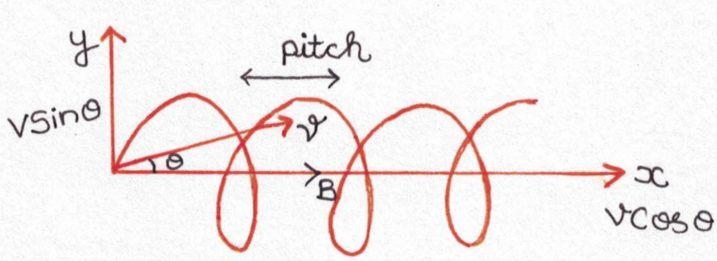
$\theta = \sin^{-1}\left(\dfrac{dqB}{mv}\right)$

★ angle b/w radii is same as that b/w tangents.

If $d > R$, angle of deviation will be 180° as shown



Case II: $(E = 0)$ & $(v \not\perp B)$



→ Due to $v\sin\theta$ path is circle.

→ Due to $v\cos\theta$ path is straight line

∴ Path taken is combination of circle and straight line i.e. "helical path".

★ In this case, we can talk about two components of velocity

① $v\cos\theta$, parallel to $\vec{B}$

② $v\sin\theta$, ⊥ to $\vec{B}$

- If there were no parallel component, the particle would have taken a circular path whose radius will be governed by the component '$v\sin\theta$'.
- However if only the parallel components were present, the particle would be translating with a constant velocity parallel to $\vec{B}$.
- In presence of both, the motion will be a superposition of the two types of motion ie particle will undergo a helical motion with radius governed by '$v\sin\theta$' & pitch governed by '$v\cos\theta$'.

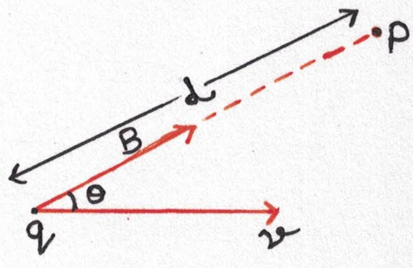
$$R = \frac{mv\sin\theta}{qB} \quad , \quad \omega = \frac{qB}{m} \quad , \quad T = \frac{2\pi m}{qB}$$

$$\left(\text{pitch} = v\cos\theta\, T \Rightarrow p = \frac{2\pi m v\cos\theta}{qB}\right)$$

Que.) An electron is accelerated through a potential diff. 'V' & launched at an angle 'θ' with the magnetic field. A target is located at a distance 'd' from the point of launching in the direction of magnetic field. Find the permissible values of 'B' so that electron may hit the target.

$$T = \frac{2\pi m}{qB}$$

$$d = np \quad ; \quad n \in I^+$$



$$d = \frac{n\, 2\pi m v\cos\theta}{qB}$$

$$B = \frac{n 2\pi m v\cos\theta}{qd}$$

$$qV = \frac{1}{2} m v^2$$

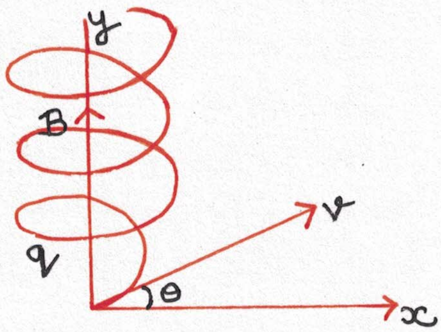
$$v = \sqrt{\frac{2qV}{m}}$$

$$B = \frac{2\pi n m}{qd}\sqrt{\frac{2qV}{m}} \cdot \cos\theta$$

$$B = \frac{2\pi n \cos\theta}{qd} \cdot m \cdot \sqrt{\frac{2eV}{m}}$$

$$B = \frac{2\pi n \cos\theta}{d}\sqrt{\frac{2mV}{e}}$$

Que.) A particle of charge 'q' is launched in the x-y plane with a speed 'v' and angle 'θ' with +ve x-axis. Magnetic field is 'B' $\hat{j}$. Find the coordinates of particle as a function of time.



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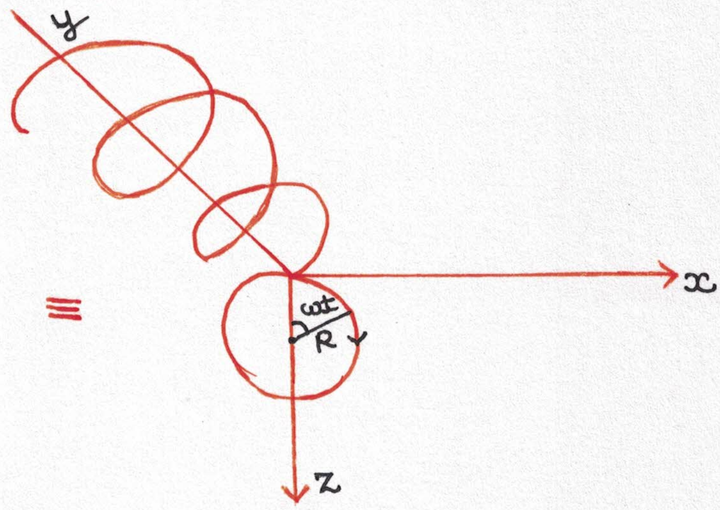


$$\beta = v\sin\theta \cdot t$$

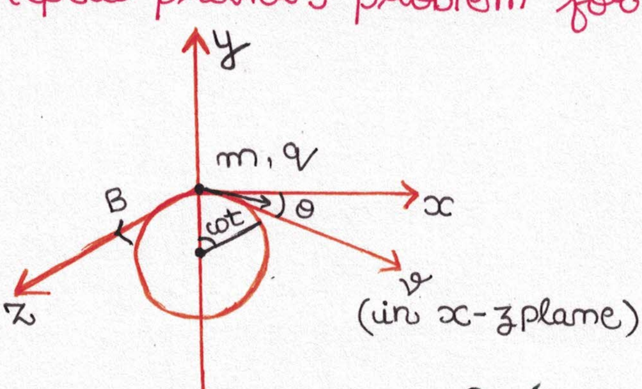
$$x = R\sin\omega t$$

$$R = \frac{mv\cos\theta}{qB} \quad , \quad \omega = \frac{qB}{m}$$

$$z = R(1-\cos\omega t)$$

$$y = v\sin\theta \cdot t$$

Que.) Repeat previous problem for the modified case.



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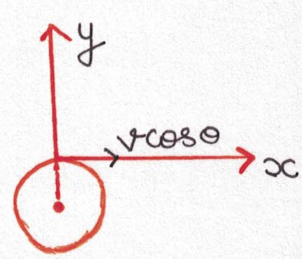


$$y = -R(1-\cos\omega t)$$

$$z = v\sin\theta\, t$$

$$x = R\sin\omega t$$

Que.) For previous problem, find velocity & particle as a function of time.

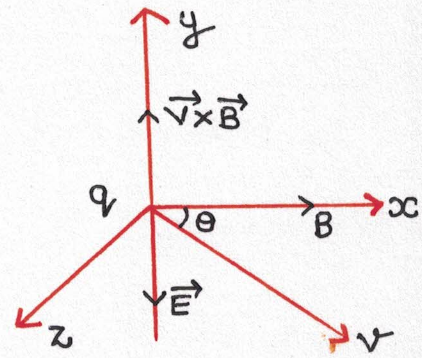
$$v_x = \frac{d}{dt}x = \omega R \cos\omega t$$

$$v_y = \frac{d}{dt}y = -\omega R \sin\omega t$$

$$v_z = \frac{d}{dt}z = v\sin\theta$$

$$\vec{v}_p = (\omega R\cos\omega t)\hat{i} - (\omega R\sin\omega t)\hat{j} + (v\sin\theta)\hat{k}$$

Que.) Find the electric field vector so that the particle can move in a straight line.



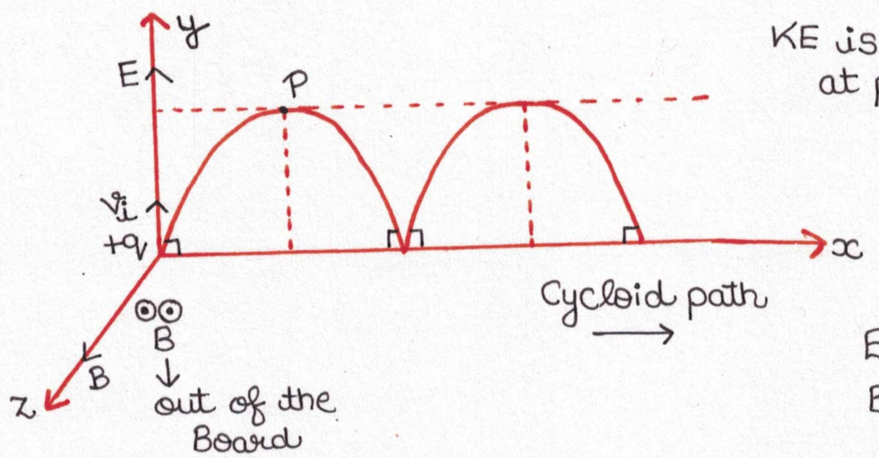
It is known that particle is moving with a constant velocity.

$$q[\vec{E} + \vec{v}\times\vec{B}] = 0$$

$$\vec{E} = -(\vec{v}\times\vec{B})$$

$$\vec{E} = vB\sin\theta(-\hat{j})$$

Case III $(v = 0)$, $(E \perp B)$



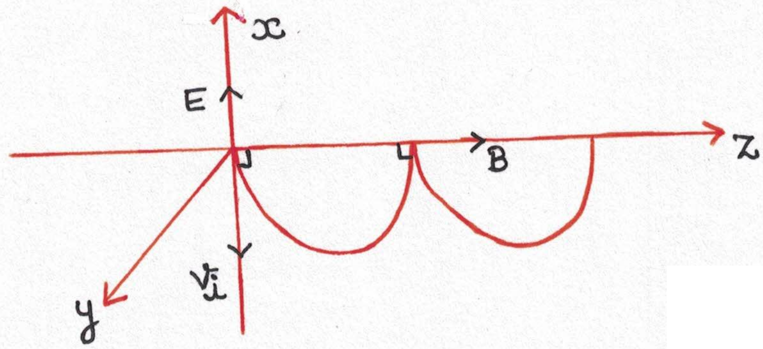
KE is max. at point P

$E = E_0\hat{j}$
$B = B_0\hat{k}$

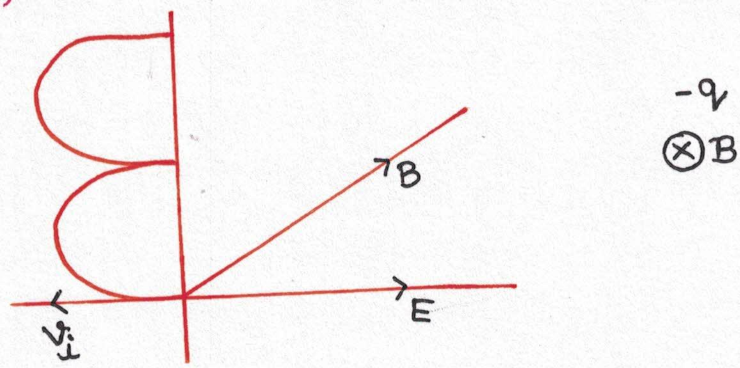
Qualitative Description:

Here initially the electric field tries to accelerate the particle in a certain direction & due to the gained velocity, magnetic field begins to curve the path. Thus, the direction of curvature can be figured out by finding the term $(q\vec{v}\times\vec{B})$. It will be mathematically shown that path is a cycloid as shown in the figure.

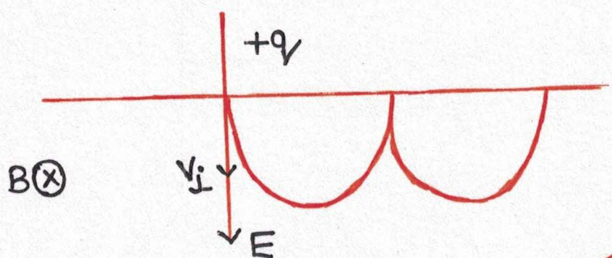
Que.) $-q$, $E = E_0\hat{i}$, $B = \odot B_0\hat{k}$. Sketch the graph.



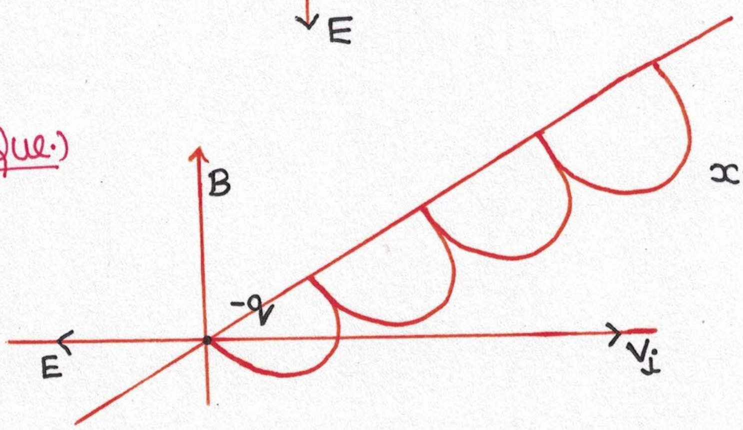
Que.)



Que.)

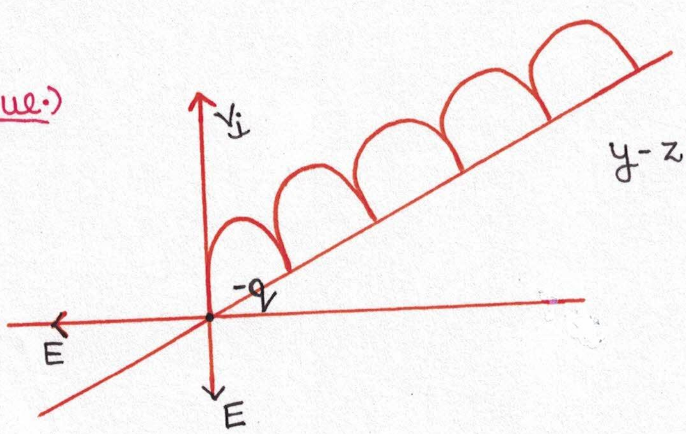


Que.)



x-z plane

Que.)



y-z plane

Quantitative Analysis:

Que.) A particle of charge 'q' & mass 'm' is released from the origin from rest under an electromagnetic field given by $\vec{E} = E_0\hat{j}$ & $\vec{B} = B_0\hat{k}$. Find

(a) velocity of the particle as a function of time.

(b) Position of the particle as a function of time.

(c) Its max. y-coordinate

(d) its max. velocity

(e) length of its intercept on x-axis.

Let velocity of the particle at any general time be

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{F} = q(\vec{v}\times\vec{B} + \vec{E})$$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q}{m}(\vec{E} + \vec{v}\times\vec{B})$$

$$\vec{v}\times\vec{B} = -B_0 v_x\hat{i} + B_0 v_y\hat{j}$$

$$\vec{a} = \frac{q}{m}\left[E_0\hat{j} + B_0 v_y\hat{i} - B_0 v_x\hat{j}\right]$$

$$a_x = \frac{dv_x}{dt} = \frac{q}{m}B_0 \cdot v_y \quad \text{— (1)}$$

$$a_y = \frac{dv_y}{dt} = \frac{q}{m}(E_0 - B_0 v_x) \quad \text{— (2)}$$

$$\frac{d}{dt}(2) = \frac{d^2v_y}{dt^2} = \frac{-qB_0}{m}\cdot\frac{dv_x}{dt}$$

$$\frac{d^2v}{dt^2} = \frac{-qB_0}{m}\cdot\frac{q}{m}B_0 v_y \qquad [\text{from eq (1)}]$$

$$\left(\frac{d^2v_y}{dt^2} = \frac{-q^2B_0^2}{m^2}v_y\right)$$

This is of the form of SHM D.eq^n

Comparing we get,

$$v_y = A\sin(\omega t + \phi) \quad \text{— (3)}$$

where $\omega = \frac{qB_0}{m}$.

We know that at $t=0$, $v_y = 0$

($\because$ particle was released from rest) and $\frac{dv_y}{dt} = \frac{E_0 q}{m}$

$a_y = A\omega \cos(\omega t + \phi)$ — (4)

$A\sin(\phi) = 0$ $(\because eq^n (3))$

$\frac{E_0 q}{m} = A\omega \cos\phi$ $(\because eq^n (4))$

$\phi = 0$ and $\left(A = \frac{E_0 q}{m\omega}\right) = \frac{E_0}{B_0}$

$\left(A = \frac{E_0}{B_0}\right)$

$v_y = A\sin(\omega t)$ — (5)

$v_{y\,max} = A = \frac{E_0}{B_0}$

where, $A = \frac{E_0}{B_0}$, $\omega = \frac{qB_0}{m}$

$\frac{dy}{dt} = A\sin\omega t$

$y = -\frac{A}{\omega}[\cos\omega t]_0^t$

$y = -\frac{A}{\omega}[\cos\omega t - \cos 0]$

$y = \frac{A}{\omega}[1 - \cos\omega t]$ — (6)

$y_{max} = \frac{2A}{\omega}$, $y_{min} = 0$

$\left(y_{max} = \frac{2mE_0}{qB_0^2}\right)$

Using (5) in (1)

$\frac{dv_x}{dt} = A\omega \sin\omega t$

$\int_0^{v_x} dv_x = A\omega \int_0^t \sin\omega t \cdot dt$

$v_x = -\frac{A\omega}{\omega}[\cos\omega t]_0^t$

$v_x = A[1 - \cos\omega t]$ — (7)

$$(v_x)_{max} = 2A$$

$$\& \quad v_{max} = 2A \qquad (\because \text{at } v_{max},\ v_y = 0)$$

Using (7)

$$\frac{dx}{dt} = A\,(1 - \cos\omega t)$$

$$x = A\left[t - \frac{\sin\omega t}{\omega}\right]_0^t$$

$$x = \frac{A}{\omega}\left[\omega t - \sin\omega t\right] \quad — (8)$$

For first, return back to x-axis,

$$\omega t = 2\pi \qquad (\text{from } (6))$$

$$\text{Intercept} = AT, \quad \text{where } T = \frac{2\pi}{\omega}$$

$$v^2 = v_x^2 + v_y^2$$

$$= A^2\sin^2\omega t + A^2 + A^2\cos^2\omega t - 2A\cos\omega t$$

$$= 2A\,(A - \cos\omega t)$$

Summary of results :

$$\omega = \frac{qB_0}{m}, \quad A = \frac{E_0}{B_0},$$

$$y_{max} = \frac{2A}{\omega}, \quad v_{max} = 2A,$$

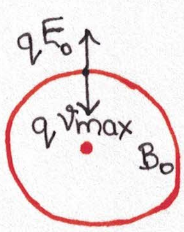
$$\text{Intercept} = AT$$

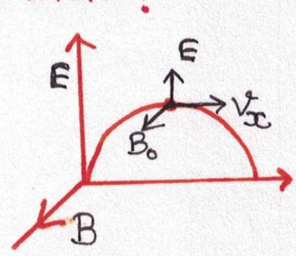
NOTE: Note that the speed of the particle for any y-coordinate can be directly calculated by work-energy theorem.

i.e. $qE_0 y = \frac{1}{2}mv^2$

Que.) In the previous problem, What will be the radius of curvature of the particle. When its y-coordinate is max. ?

$$F = -q\,(E_0\hat{j} - B_0 \underset{\substack{\downarrow \\ v_{max}}}{v_x}\hat{j})$$





$$a = \frac{F_c}{m}$$

$$= \frac{qB_0v_{max} - qE_0}{m}$$

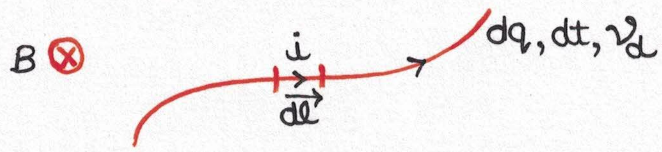
But $a = \frac{v_{max}^2}{R}$

$$R = \frac{m v_{max}^2}{qB_0 v_{max} - qE_0}$$

$$R = \frac{4A^2 m}{2qB_0 A - qE_0} = \frac{4E_0 m}{qB_0^2}$$

MAGNETIC FORCE ON A CURRENT CARRYING WIRE

Consider an element of current carrying wire having length 'dl' through which a 'dq' charge passes in time 'dt'. Let



v_d be the drift velocity then we can calculate the Lorentz Force on the element as

$$d\vec{F} = (dq)\, \vec{v_d} \times \vec{B}$$

$$d\vec{F} = (dq)\, \frac{d\vec{l}}{dt} \times \vec{B}$$

$$\left(d\vec{F} = i\, d\vec{l} \times \vec{B}\right)$$

To find the force on the entire wire we can integrate the above result, ie

$$\vec{F} = \int (i\, d\vec{l}) \times \vec{B}$$

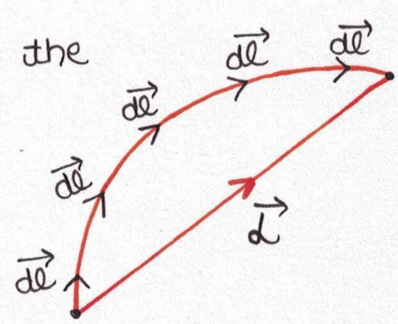
★ A 'Special Case' of this result occurs when the magnetic field is uniform.

In that case,

$$\vec{F} = i \left(\int d\vec{l}\right) \times \vec{B}$$

$$\vec{F} = i\vec{L} \times \vec{B}$$

where $\vec{L}$ is the vector joining the initial & final point of the wire.



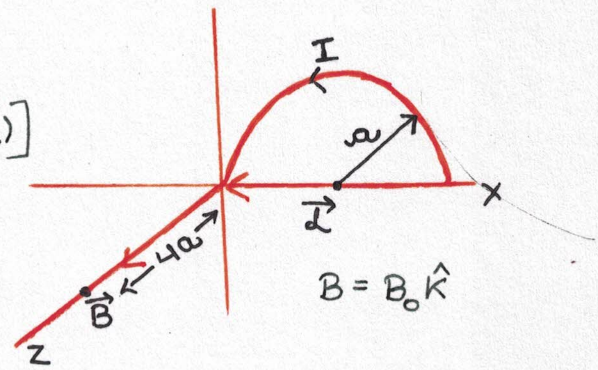
NOTE: In uniform magnetic field the net force on a current carrying wire only depends on an initial and final points of the wire.
Thus, Net **force** on any closed loop will be zero.

Que.) Find the net magnetic force on the segment of a current carrying wire shown in fig.

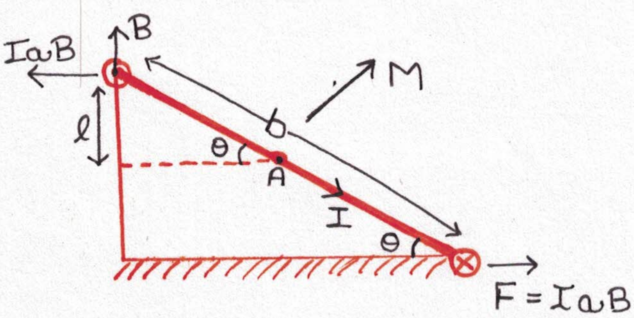
$$\vec{L} = -2a\hat{i} + 4a\hat{k}$$

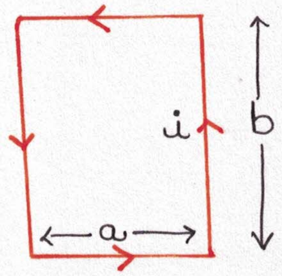
$$\vec{F} = I\left[(-2a\hat{i} + 4a\hat{k}) \times (B_0\hat{k})\right]$$

$$\vec{F} = 2aIB_0(+\hat{j})$$



TORQUE ON A CURRENT-CARRYING LOOP





FRONT VIEW

$$\tau_A = (\underbrace{IaB}_{F} \times \underbrace{\frac{b\sin\theta}{2}}_{l}) \times 2$$

$$\tau = I\underbrace{ab}_{\text{area}}B\sin\theta$$

$$\tau = MB\sin\theta$$

$$(\vec{\tau} = \vec{M} \times \vec{B})$$

NOTE: 1.) We have proved the result for a rectangular coil but it is true for coil of any shape, which can be proved by making very small rectangles.

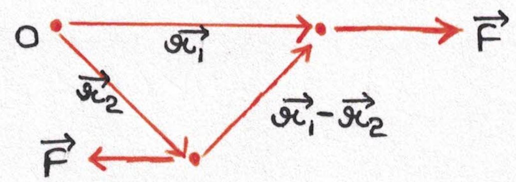
NOTE: 2.) For a coil of 'N' no. of turns, the torque

$$\tau = NIAB\sin\theta$$

(A → area)

$$= N \text{ (torque on a single loop)}$$

3.) Even though we have found the torque about the central axis of the coil, the same result holds for the torque about any point in the universe. This is because the torque of a couple, (two equal & opposite forces) is independent of choice of point as shown.

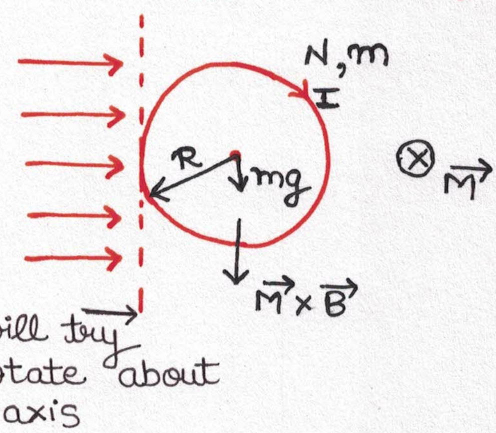


invariant for given two points of application of force.

Que.) Coil of 'N' turns lying on a horizontal table. What is the min. value of B if we want the loop to lift off the table?

$$NI \cdot \pi R^2 B \sin 90^\circ = mgR$$

$$B = \frac{mg}{NI\pi R}$$

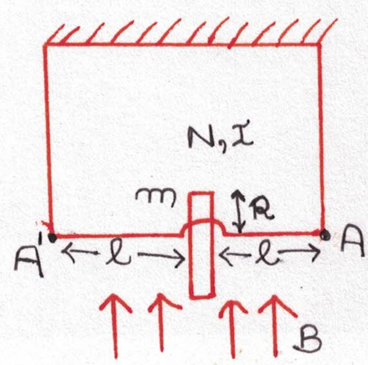


Que.) Circular loop mounted on a massless shaft. What should be the min. value of 'I' if we want one of the strings to just become loose.

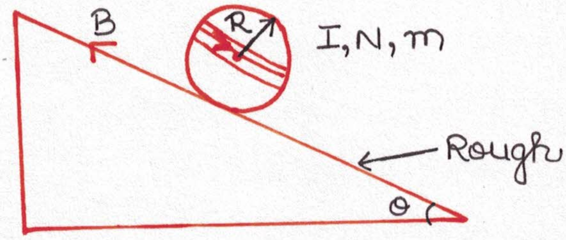
$$\tau_A = 0$$

$$NI\pi R^2 B = mgl$$

$$I = \frac{mgl}{NB\pi R^2}$$



Que.) Find min. value of Magnetic field so that sphere remains in equilibrium ie to prevent rolling of sphere.



$$mg\sin\theta\,(R) = NI\pi R^2 B\sin\phi$$

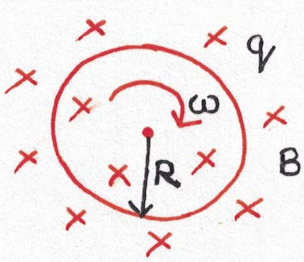
$$B = \frac{mg\sin\theta}{\pi NIR\sin\phi}$$

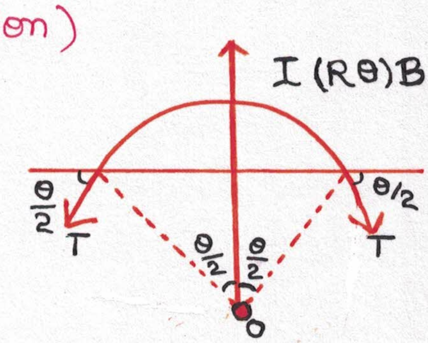
For B_{min}, $\sin\phi = 1$

$$B_{min} = \frac{mg\sin\theta}{\pi INR} \quad \left(\begin{array}{c}\text{antiparallel}\\ \text{to surface}\end{array}\right)$$

Que.) Ring Carrying charge 'q'. Find max. permissible 'ω' so that the ring does not break.

(T_0 = max. permissible tension)





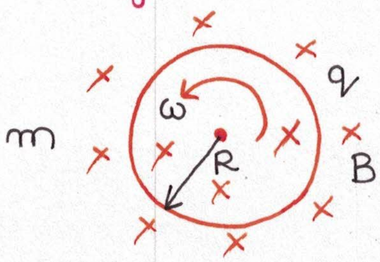
$$2T_0\sin(\theta/2) = \underset{\downarrow\ \frac{q\omega}{2\pi}}{I}R\theta B$$

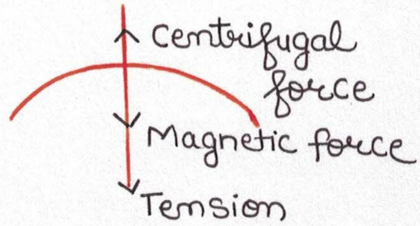
$$\omega = \lim_{\theta\to 0}\frac{4\pi T_0\sin(\theta/2)}{2qRB\,\theta/2}$$

$$\omega = \frac{2\pi T_0}{RBq}$$

→ If the ring had not been massless, max permissible 'ω' will decrease.

Que.) Find value of 'ω' for which ring is at the verge of breaking.





$$m\omega^2 R = \frac{q\omega RB}{2\pi} + 2T_0$$

permissible 'ω' would increase.

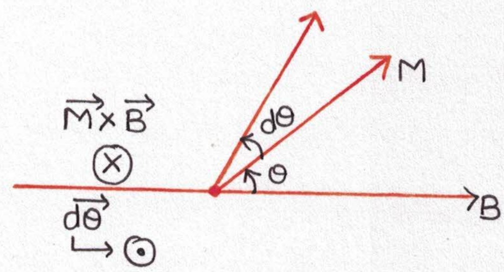
POTENTIAL ENERGY OF A CURRENT CARRYING LOOP IN A MAGNETIC FIELD

$$dw = \vec{\tau} \cdot d\vec{\theta}$$

$$\tau = MB\sin\theta$$

$$dw = \tau\, d\theta \cos 180^\circ$$

$$= -MB\sin\theta . d\theta$$



$$du = dw$$

$$dU = MB\sin\theta . d\theta$$

$$U = -MB\cos\theta + C$$

For convenience, we choose $C = 0$

$$\left(U = -\vec{M}\cdot\vec{B}\right)$$

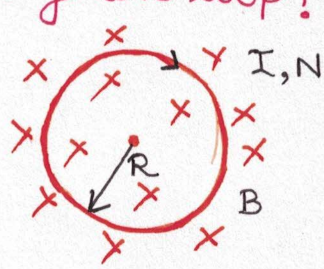
U will be max. for $\theta = 180^\circ$

ie $\vec{M}$ & $\vec{B}$ are anti-parallel.

Que.) What is the work done in flipping the loop?

$$W = 2MB$$

$$= 2B\,IN\pi R^2 B$$



$$U_i = -MB\cos 0^\circ = -MB$$

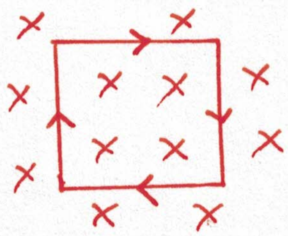
$$U_f = -MB\cos 180^\circ = MB$$

$$U_i - U_f = -2MB$$

$$W = -\Delta U$$

$$W = 2MB$$

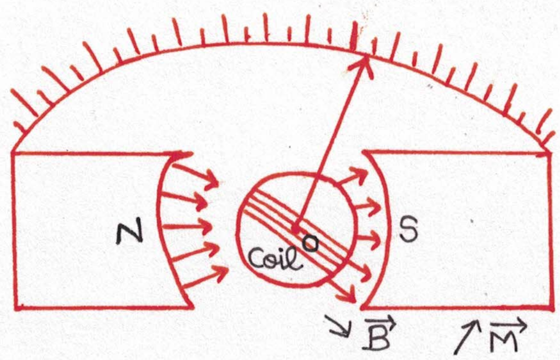
Que) What will happen to potential energy as it tries to attain Circular shape.

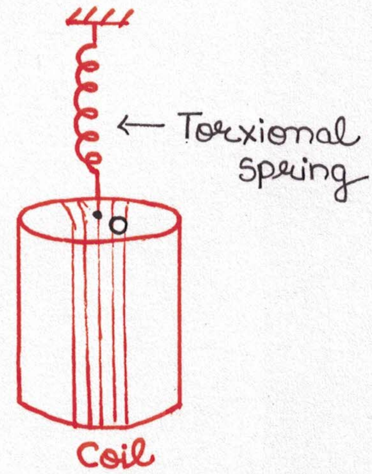


$$2R_s = 2\pi r_e$$
$$\pi r^2 \qquad R^2$$

U will decrease.

MOVING - COIL GALVANOMETER





For equilibrium

Magnetic Torque = Spring Torque

★ This is only an assumption that $\vec{B}$ is radial for simple calculations.

$$NIAB \sin 90^\circ = k\theta$$

↓ Torxional spring constant

$$\left(I = \frac{k\theta}{NAB} \right)$$

CURRENT SENSITIVITY

$$\text{Current sensitivity} = \frac{d\theta}{dI}$$
$$= \frac{NAB}{k}$$

★ If sensitivity ↑, range ↓